Sifted Disks

reducing the number of sample points
retaining randomness
improving quality

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7!

Eurographics 2013

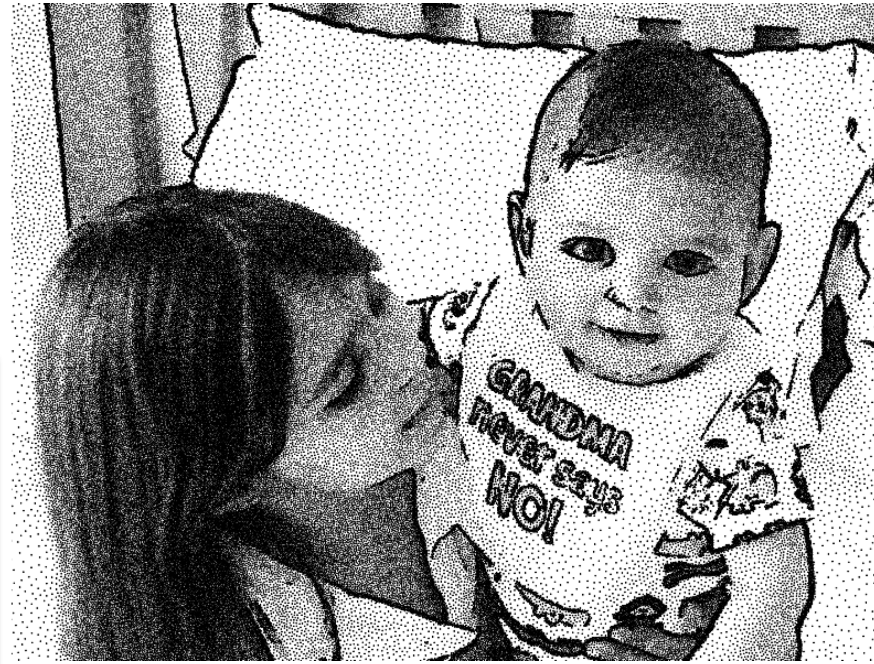
presenter = Scott


Mohamed Ebeida's (first author's) kids, in full color
Omar
Meera
GRANDMA
never says
NO!

Application

1. grayscale -> sizing function for

2. Stippling via Maximal Poisson-Disk Sampling



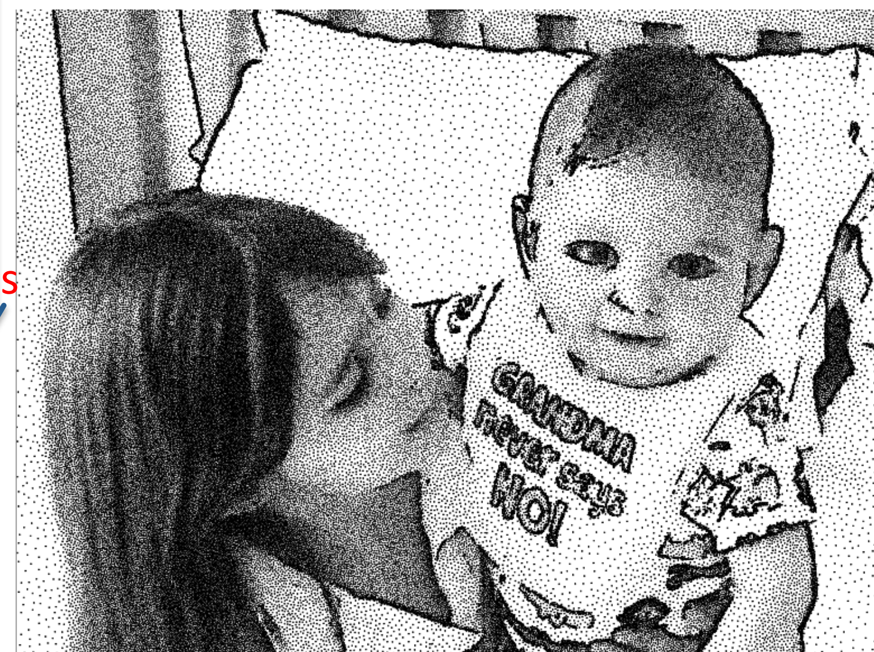
3.
Sift points

Replace 2 for 1.

Respect original sizing function.

Fewer points
Minimal quality loss

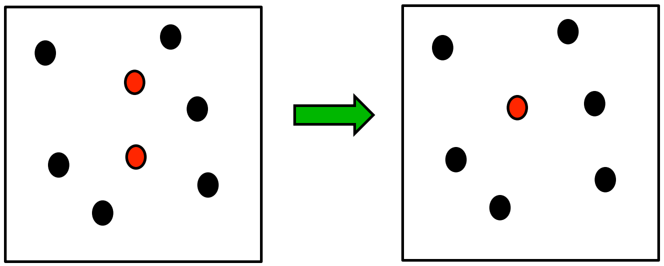
edge-detect



Universally lighter, but features still distinct

Overview

- Input: point sample distribution
  Poisson disks, Delaunay Refinement
  - Sizing function
    - Adheres approximately
- Observe: other distributions also respect sizing function,
  might be smaller
- Process
  - Replace points 2-for-1
  - Adhere to sizing function
- Result
  - Fewer points --- how many?
  - Retained randomness --- surprise!



Mesh Improvement

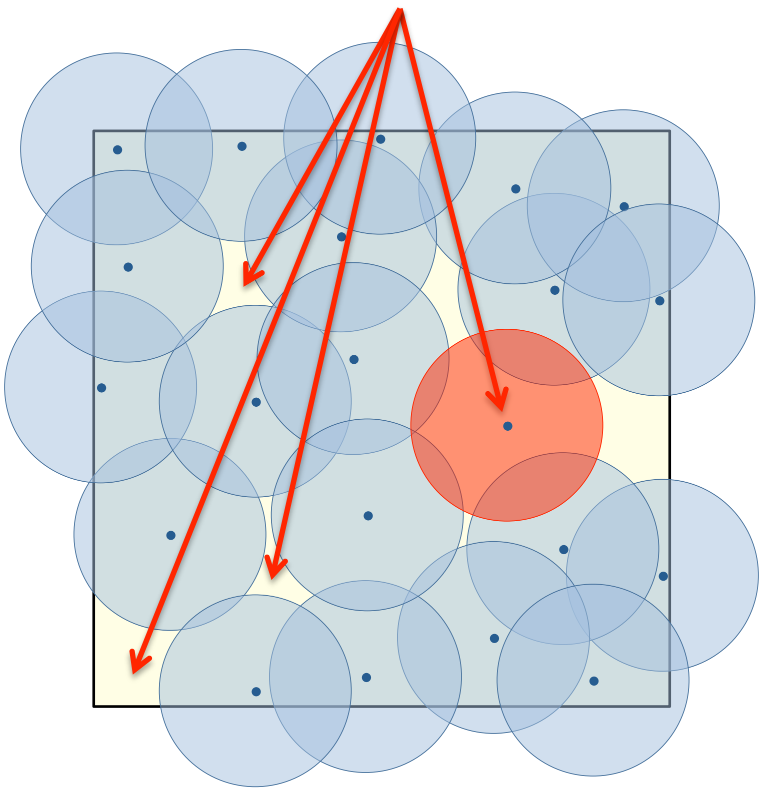
Sifting triangulations from

DR Delaunay Refinement

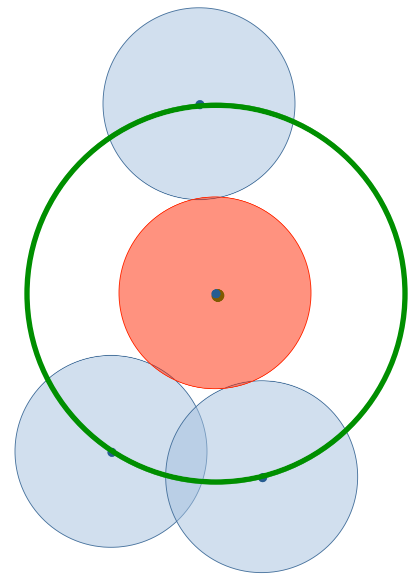
ODR Delaunay Refinement w/ Off-centers

MPS Maximal Poisson-disk Sampling

MPS
new disk
global uniform random locations
outside prior disks

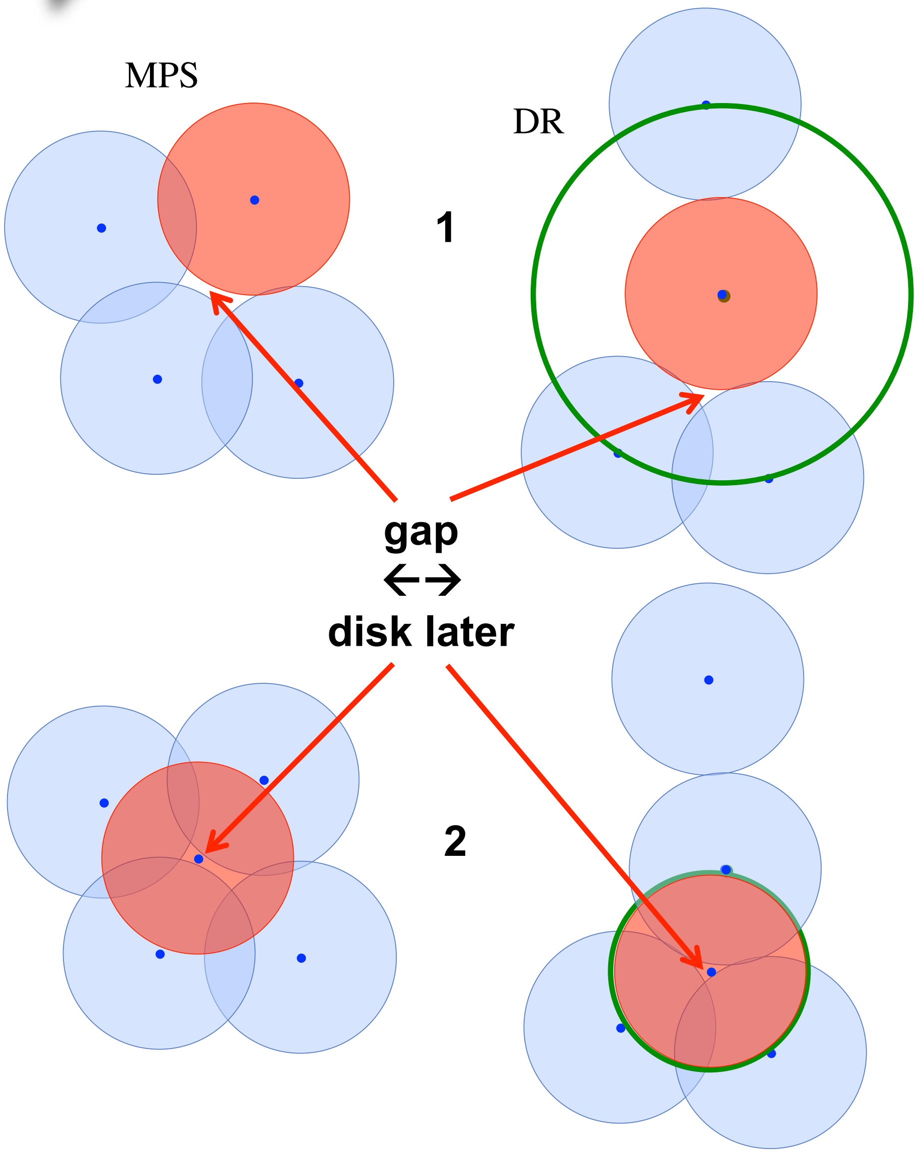


DR
new disk
center of large empty dual circle



Problem

Painting Yourself Into a Corner



MPS, DR

easy to introduce a small gap that later forces

- **distance = r + eps**
- **dense sampling**

lots of r ≈1 edges

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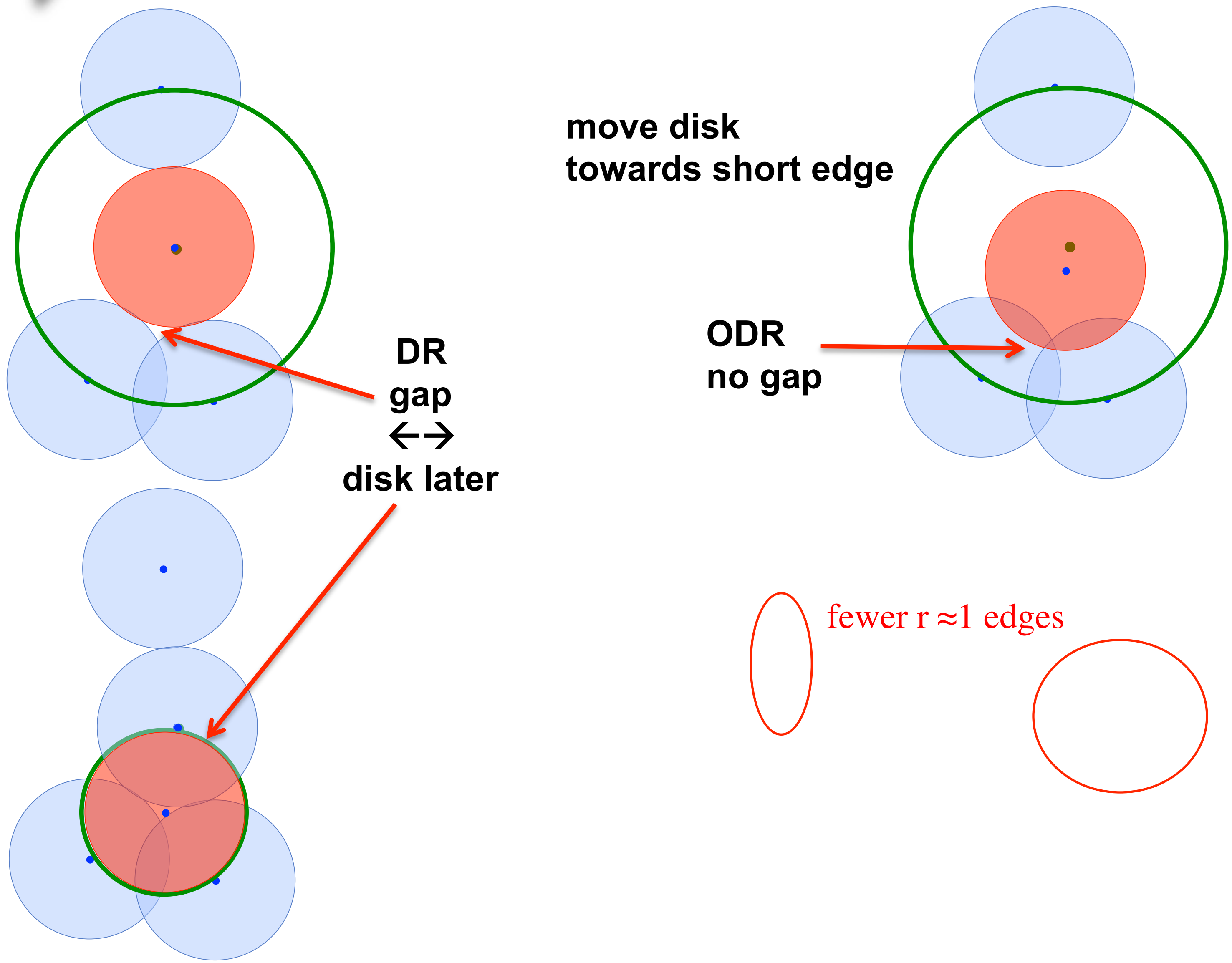
DR Solution
off-centers DR (ODR)

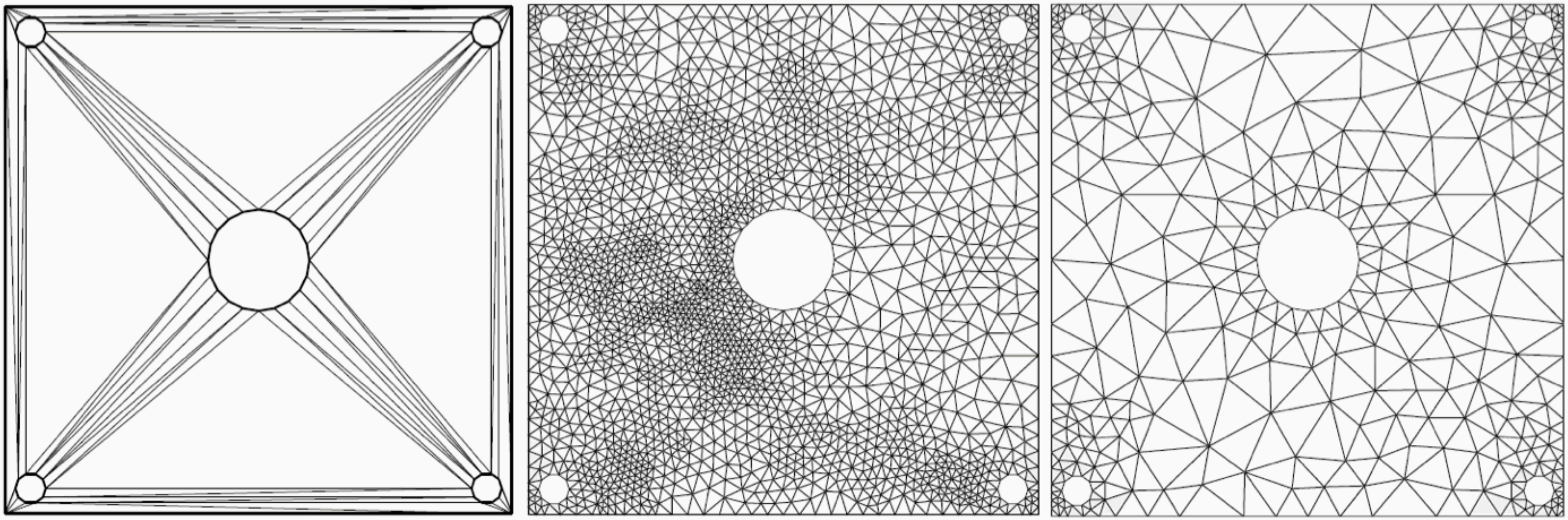


Offcenters reduces density…
by a lot for non-uniform sizing functions

But we will focus on
r = 1, uniform

10 Alper Ungör

"Off-centers: A new type of Steiner points for computing size-optimal quality-guaranteed Delaunay triangulations"

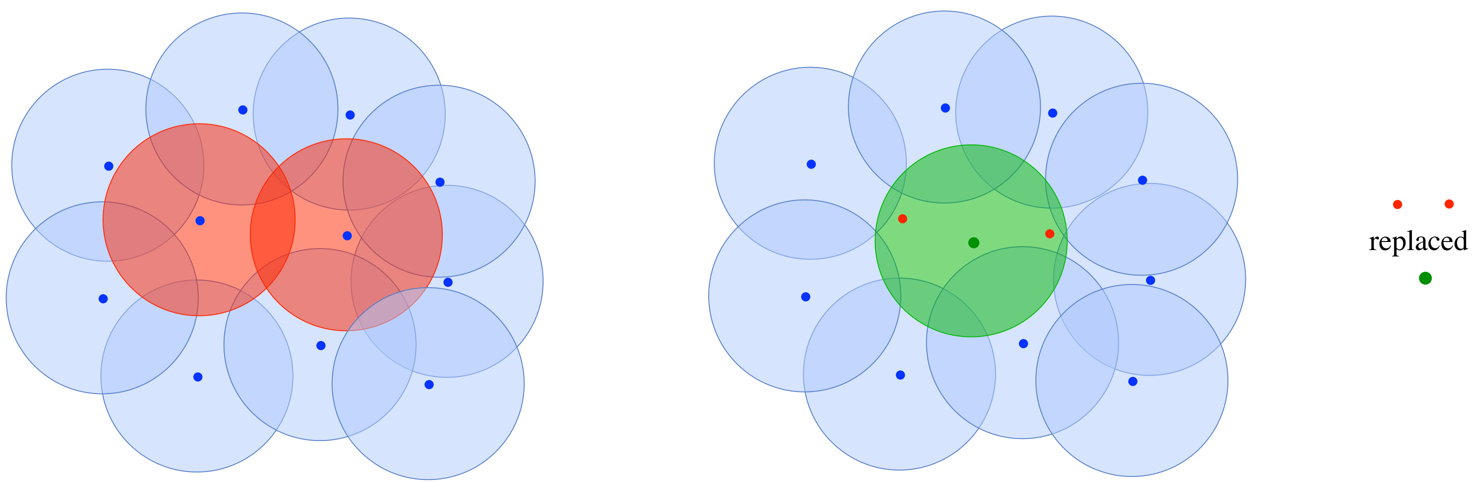


(a) input (b) DR (c) ODR

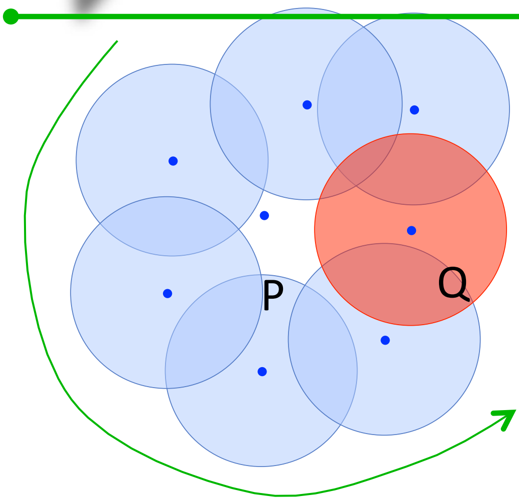
Fig. 4. Input PSLG is a plate with five holes described by 64 points and 64 segments. Smallest angle in the initial triangulation (a) is about 1°. Smallest angle in both output triangulations is 34°. Circumcenter insertion (`triangle` software) introduces 1984 Steiner points resulting a mesh with 3910 triangles (b). Off-center insertion introduces only 305 Steiner points resulting a mesh with 601 triangles (c).

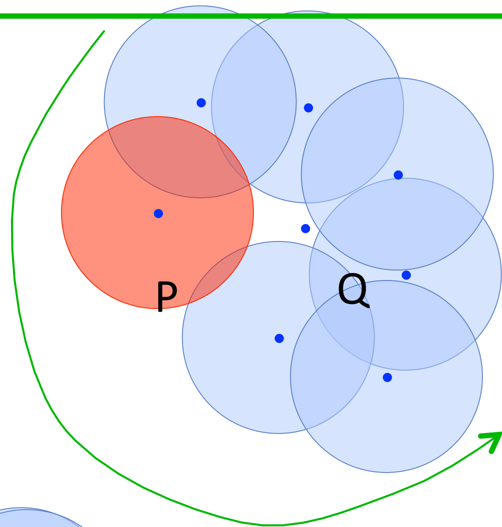
Our MPS-like Solution: Sifting

- Post-process
- For all pairs of points with overlapping disks
  - Try to replace 2-for-1
  - (Replacing changes the set of overlapping pairs)
- Quit when no pair can be replaced



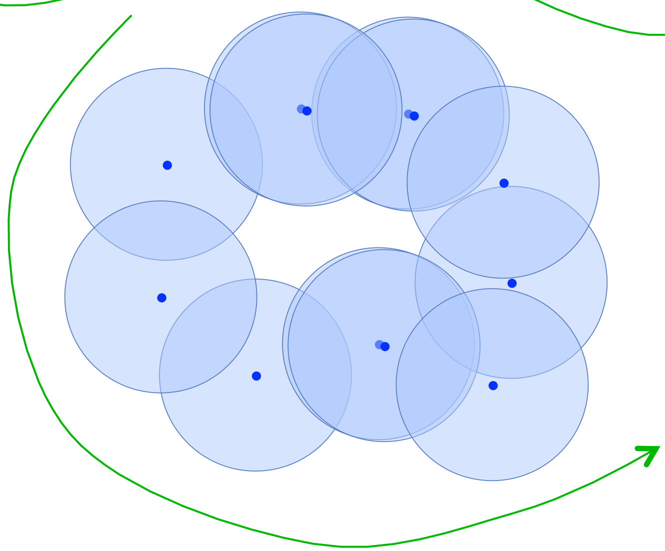
Sifting Algorithm
Gather Boundary Disks



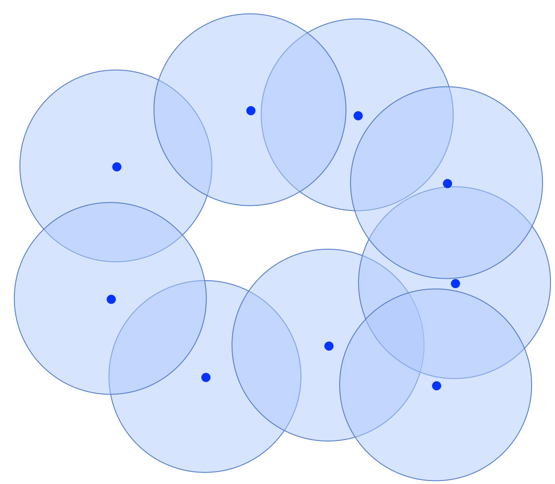


1. Gather disks overlapping P
- **(Q)**

Sort by angle around P (Q)



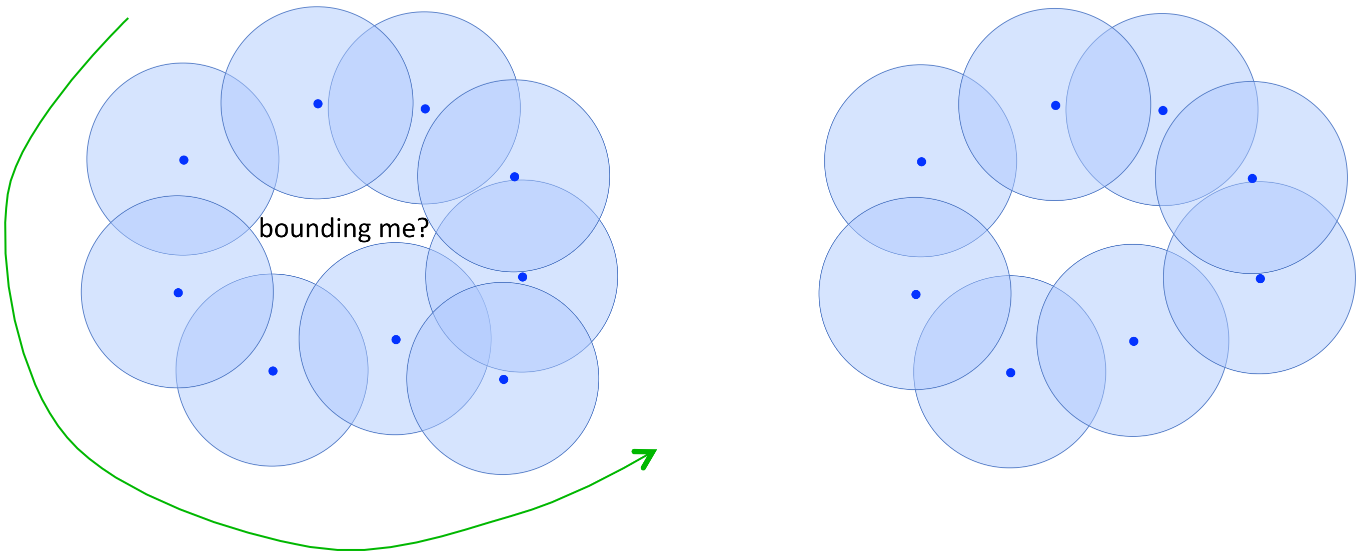
2. Stitch lists together
- **Replace Q in ListP by ListQ**

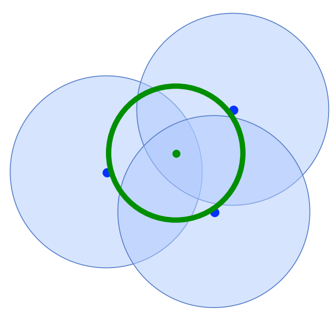


3. Remove duplicate disks

Sifting Algorithm
Winnow Non-Bounding Disks

- **Remove disks not bounding the white area**
  - **Test consecutive disks in list, see if left point of intersection is inside next disk**

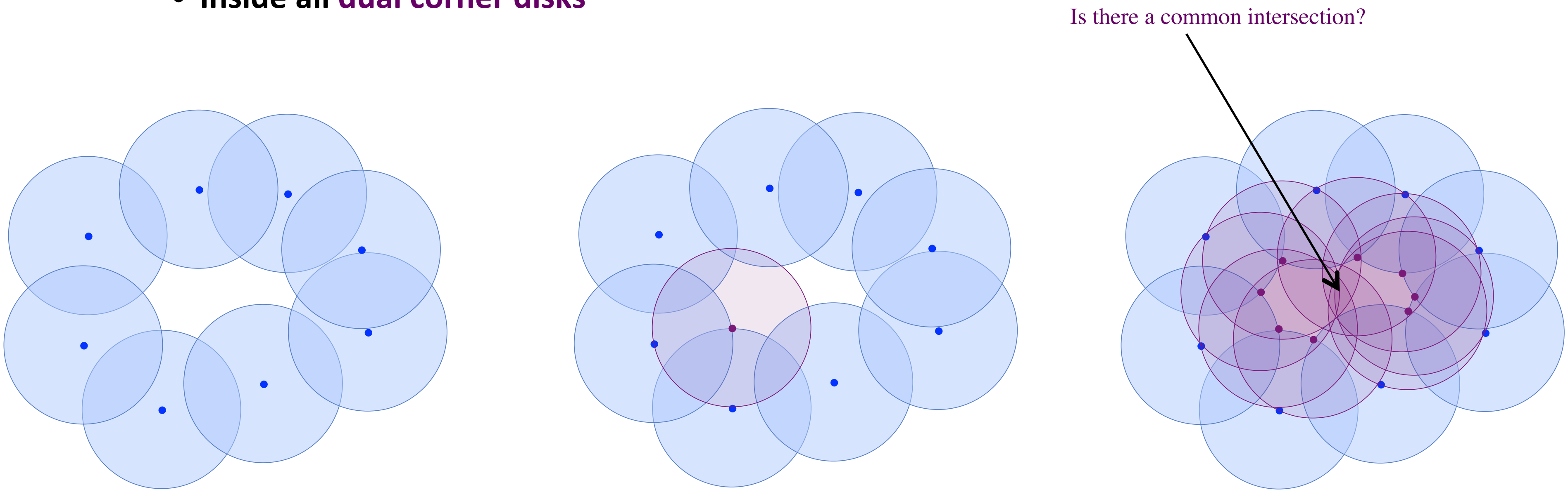




  - **In-circle test speeds up intersection point test by 3x**
    technical for non-constant radius, details in paper ☺

Sifting Algorithm
Exclusion – Inclusion Disks

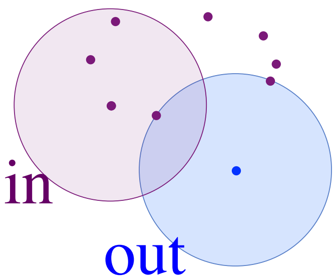
- A new disk will cover all the white area
  - Iff it covers all the corners of intersection
- Reason: because disks are convex
- Need replacement disk
  - Outside all sample disks
  - Inside all dual corner disks



Sifting Algorithm

Search for Random Location – Using "Simple MPS"

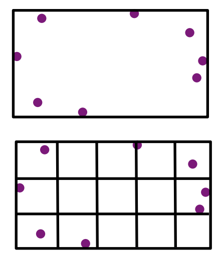
- Problem: find random point that is
  - Outside all sample disks
  - Inside all dual corner disks

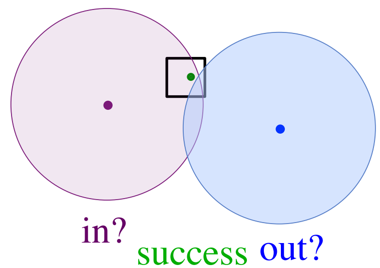


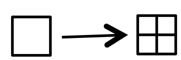
Solution:
- Simple MPS [Ebeida et al. Eurographics 2012]
- extended for purple inclusion disks

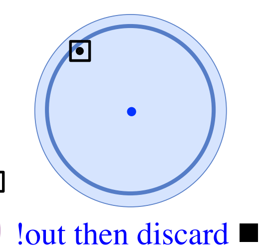
Flat quadtree
- Keep / discard squares entirely inside / outside disks
- Sample from kept squares – done if success
- Refine all squares and repeat

If last square is discarded (machine precision)
- No replacement disk exists, try a different pair









Simple MPS Algorithm Details

Initialize
  bounding box of purple corners
  subdivide into squares - diagonal about radius

Sample C | #square times |
  pick a square
  pick a point p in the square
  keep p if out-blue & in-purple
   success!

Refine all squares
  center inside a blue circle - delta? Discard
  center outside a purple circle + delta? Discard

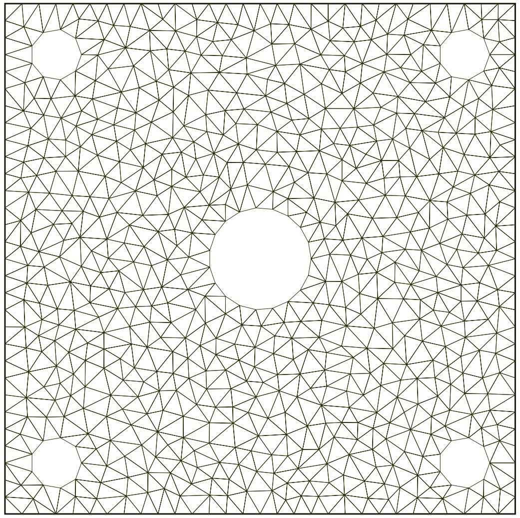
Repeat with refined squares

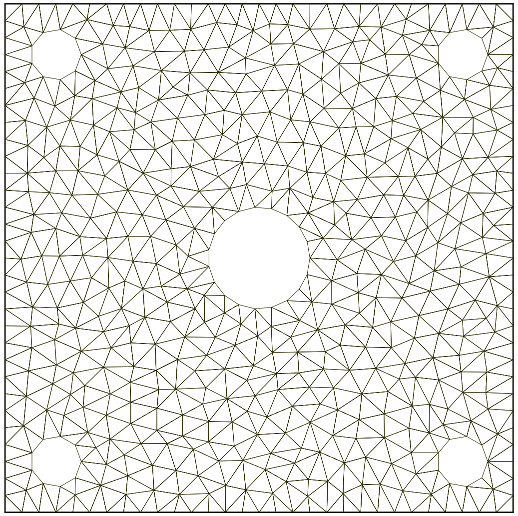
No squares? No replacement exists

Sifting Improves All Uniform Test Distributions

- **Sifting improves, MPS, DR and further improves ODR**

MPS

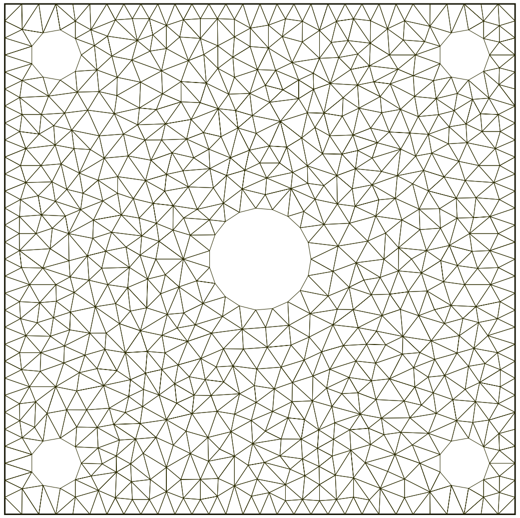


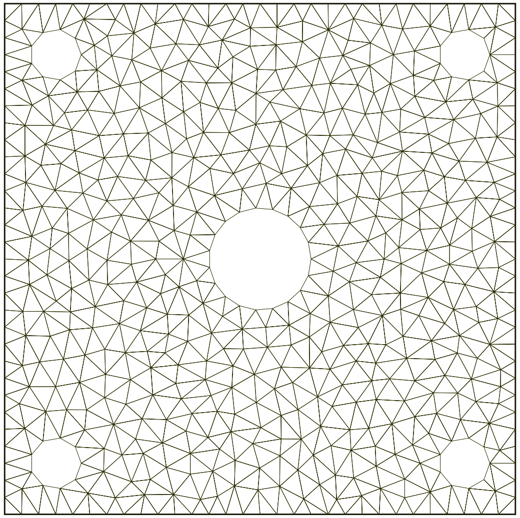


sMPS

Sift->

DR





sDR

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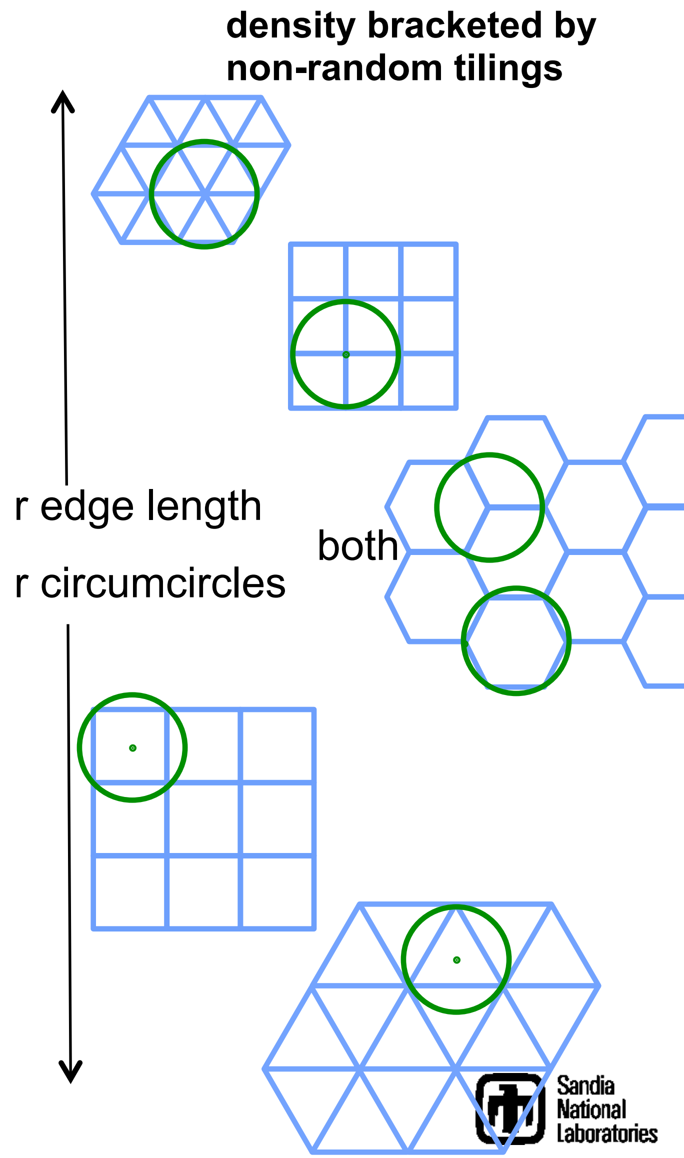
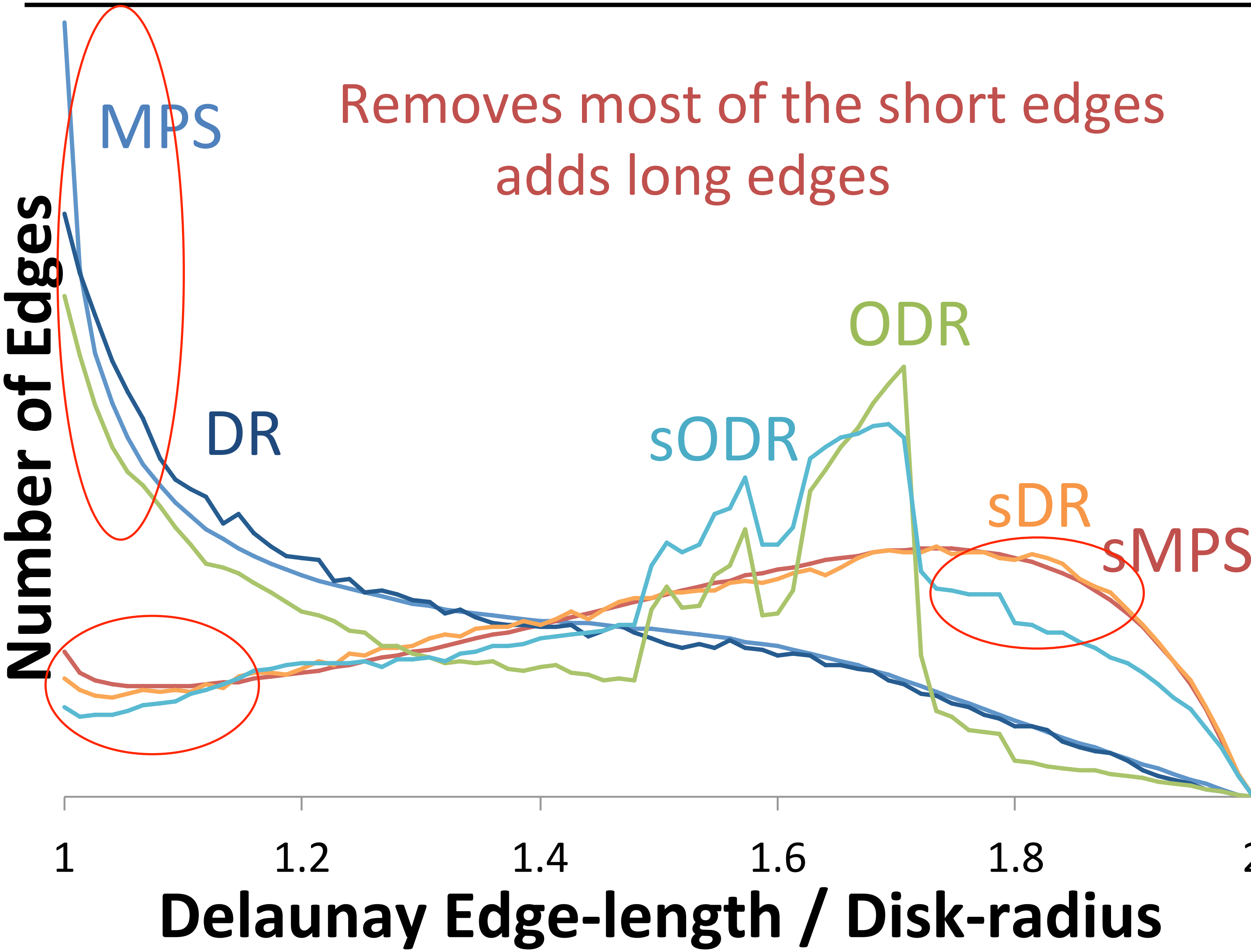
Sifting reduces number of points by ≈25%

	sample type	point density	relative density	Delaunay edge lengths
densest possible	$\triangle(r)$	$\frac{2}{\sqrt{3}} r^{-2}$	3	$\{r\}$
	$\square(r)$	r^{-2}	2.60	$\{r, \sqrt{2}r\}$
	$\hexagon(r)$	$\frac{4}{3\sqrt{3}} r^{-2}$	2	$\{r, \sqrt{3}r, 2r\}$
input	DR(r)	$0.75\, r^{-2}$	1.95	$[r, 2r)$
	MPS(r)	$0.70\, r^{-2}$	1.82	$[r, 2r)$
	ODR(r)	$0.64\, r^{-2}$	1.66	$[r, 2r)$
sifted	sDR(r)	$0.57\, r^{-2}$	1.48	$[r, 2r)$
	sMPS(r)	$0.51\, r^{-2}$	1.33	$[r, 2r)$
	sODR(r)	$0.51\, r^{-2}$	1.33	$[r, 2r)$
	$\square(\sqrt{2}r)$	$\frac{1}{2} r^{-2}$	1.30	$\{\sqrt{2}r, 2r\}$
sparsest possible	$\triangle(\sqrt{3}r)$	$\frac{2}{3\sqrt{3}} r^{-2}$	1	$\{\sqrt{3}r\}$

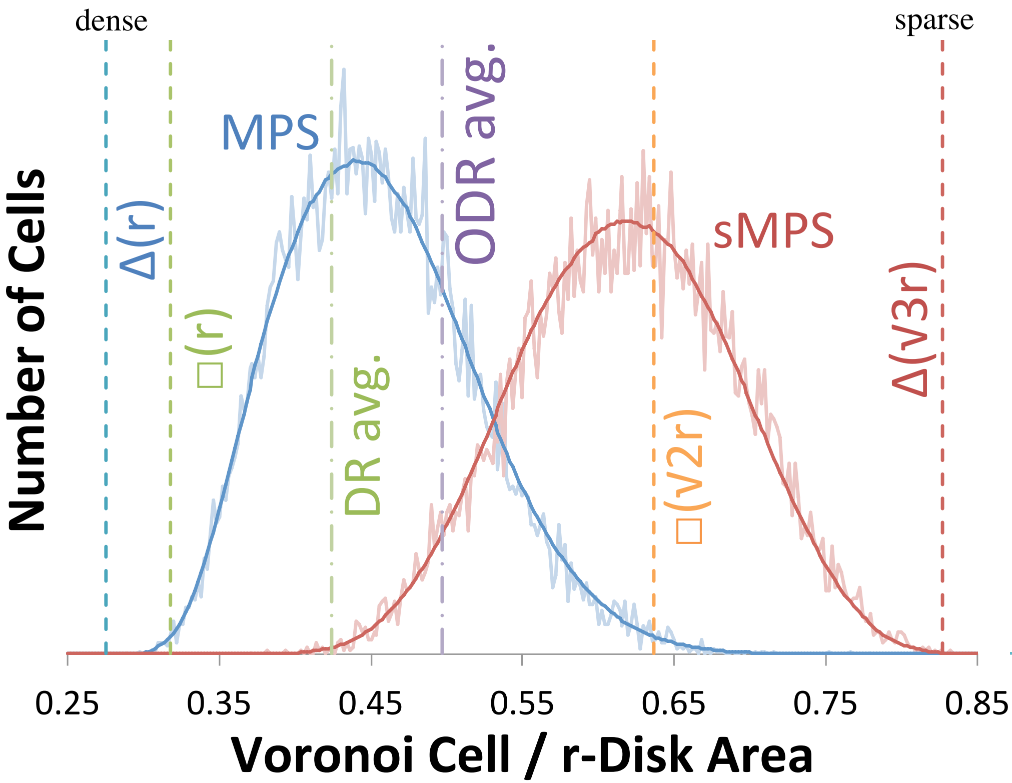


Sifting changes triangulation edge lengths, angles, Voronoi cell squish

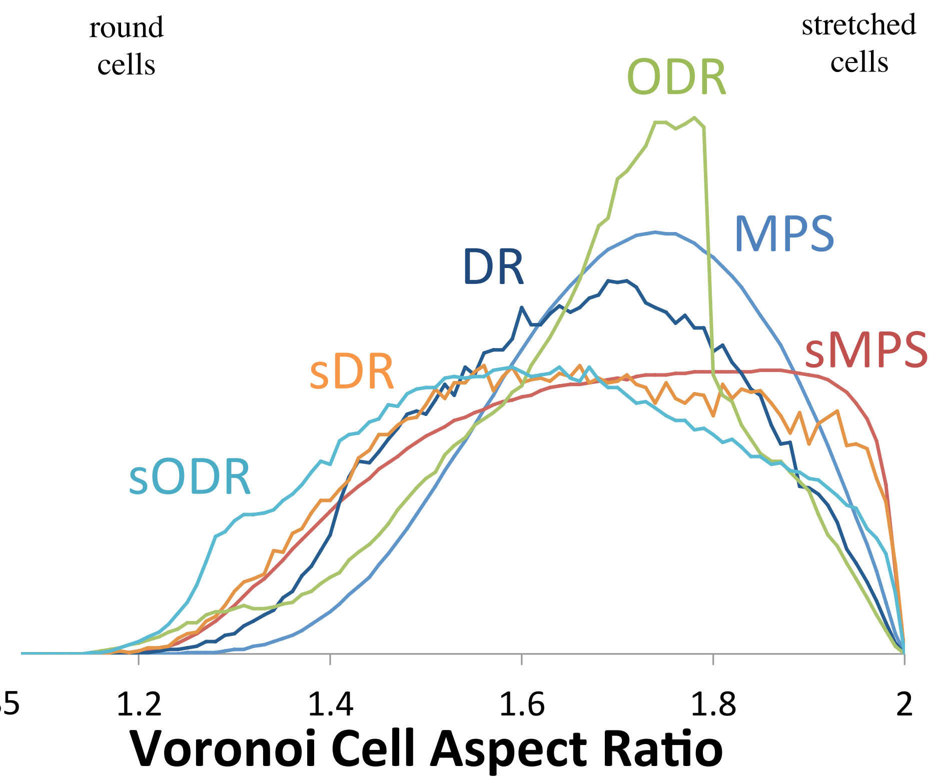


Sifting changes triangulation edge lengths, angles, Voronoi cell squish

shifts Voronoi cell area distribution



spreads Voronoi cell aspect ratio

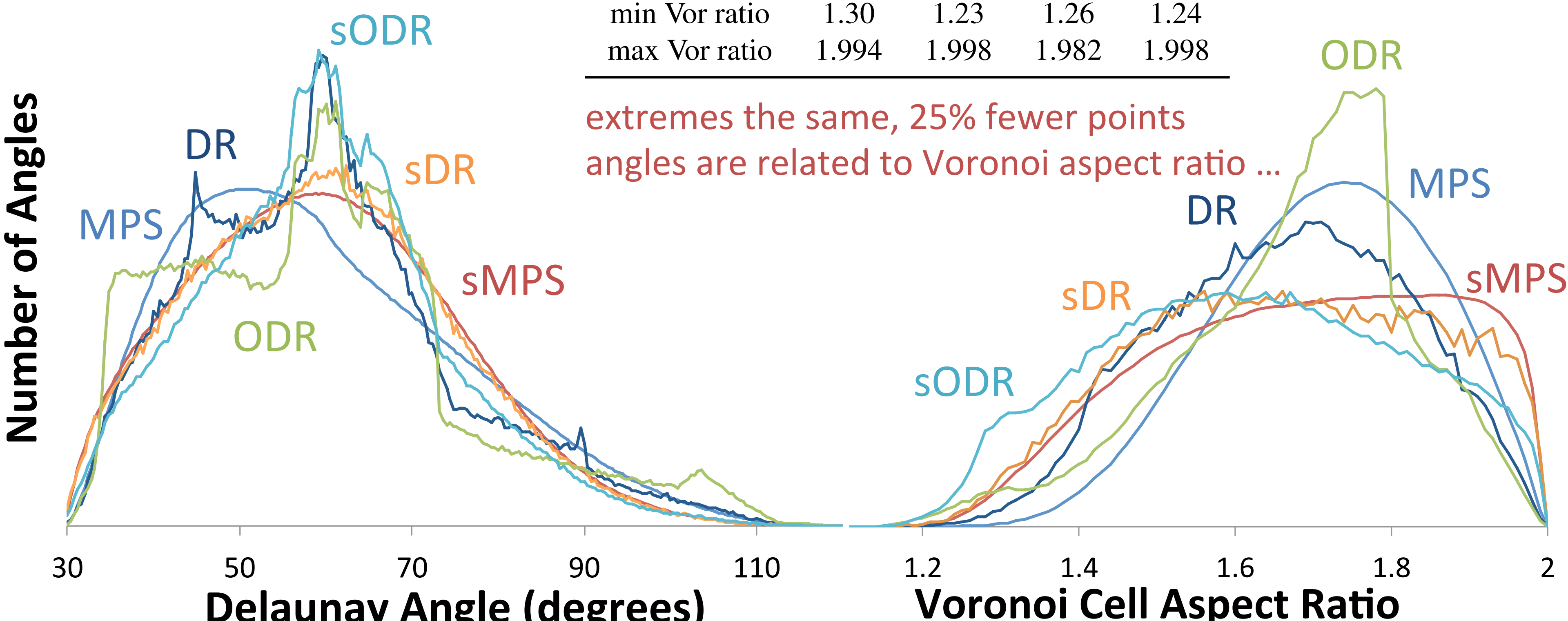


Sifting changes triangulation edge lengths, angles, Voronoi cell squish

	MPS	sMPS	DR	sDR
interior points	580	419	580	417
…reduction	-	27%	-	28%
min angle	30.6	30.5	31.7	30.6
max angle	115.5	114.7	110.7	115.5
min Vor ratio	1.30	1.23	1.26	1.24
max Vor ratio	1.994	1.998	1.982	1.998

changes angle distributions

extremes the same, 25% fewer points
angles are related to Voronoi aspect ratio …



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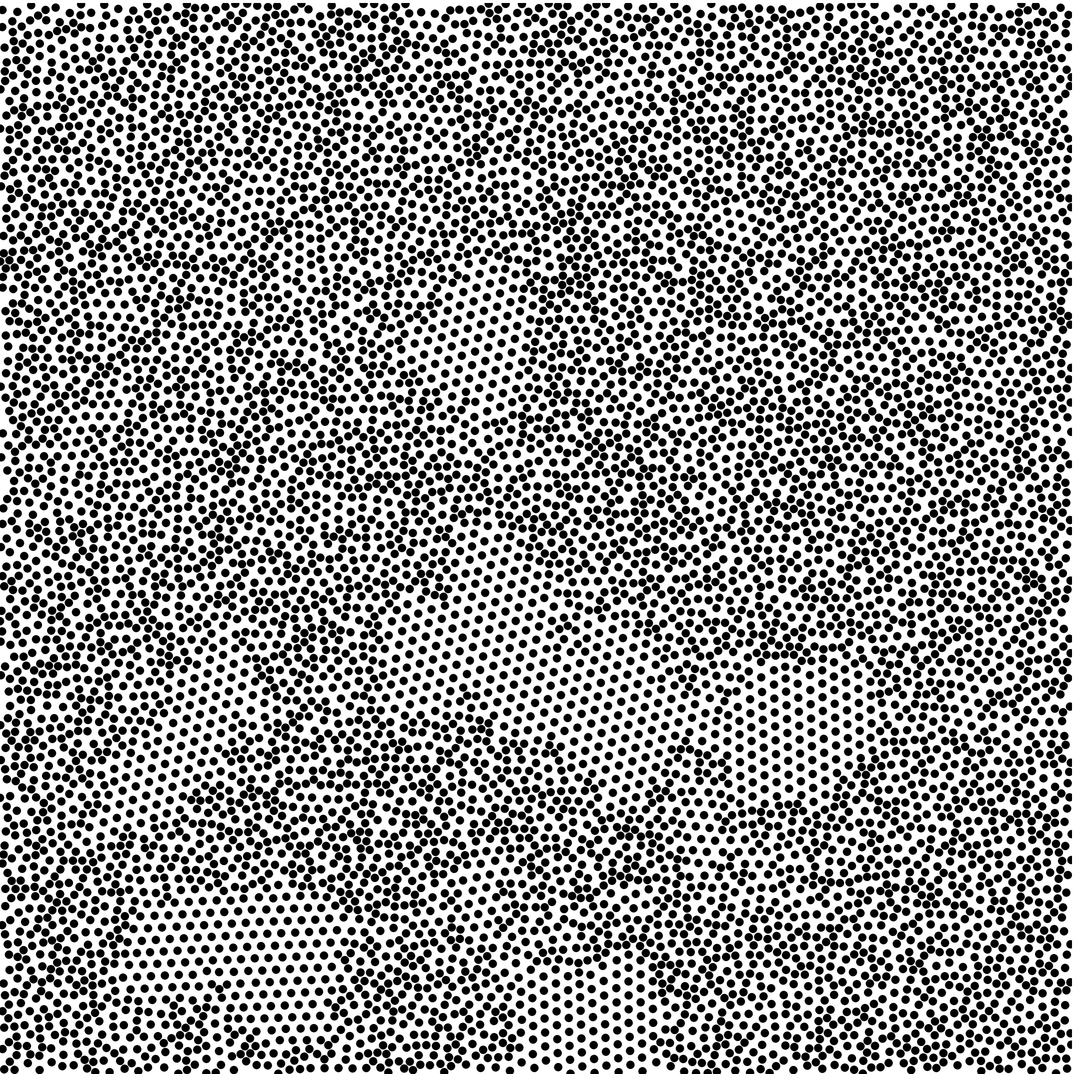
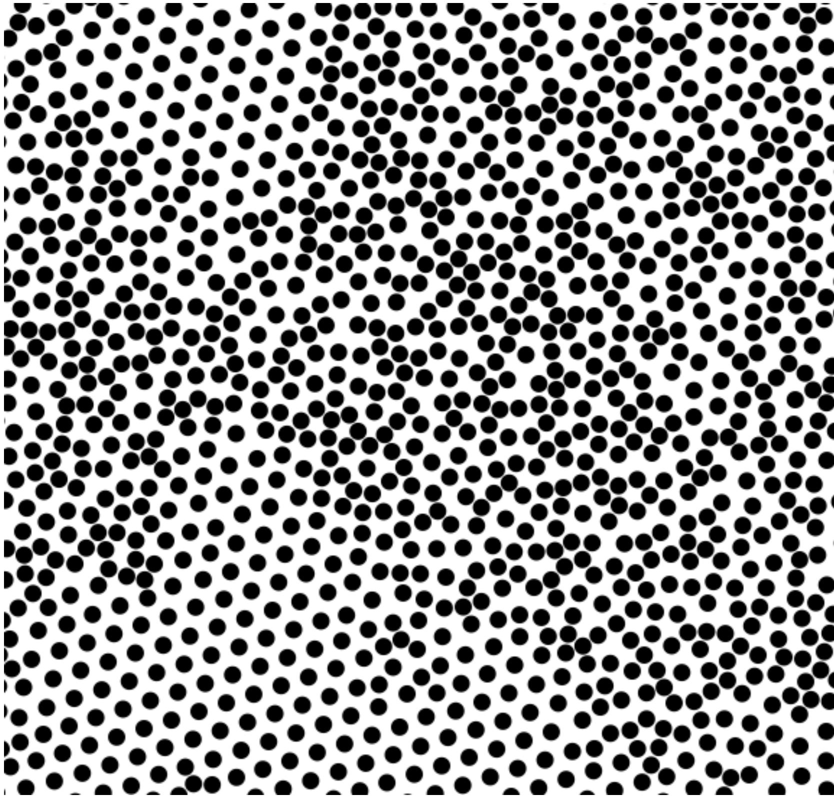
DR and ODR
Sometimes Appear Random

- **Many control parameters**
  - **Which circle (off) center to insert next?**



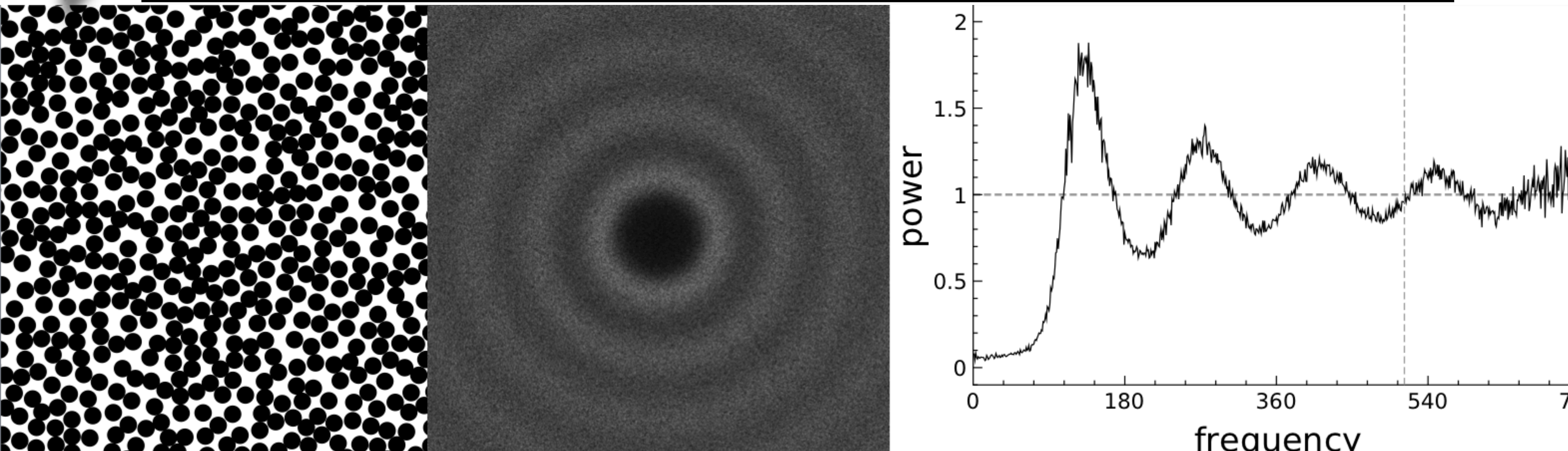
We picked random-looking versions for comparisons, Not these!



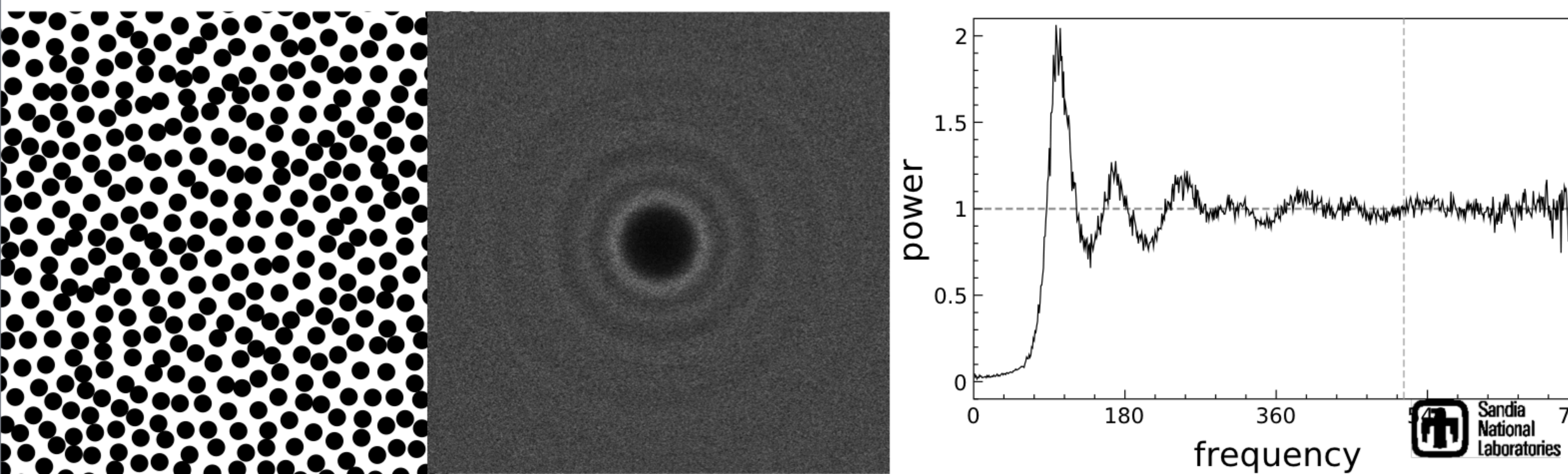
ODR

Sifting Retains Randomness
Surprise! But not identical.

MPS



Can you tell me which is “better”?
What’s ideal?



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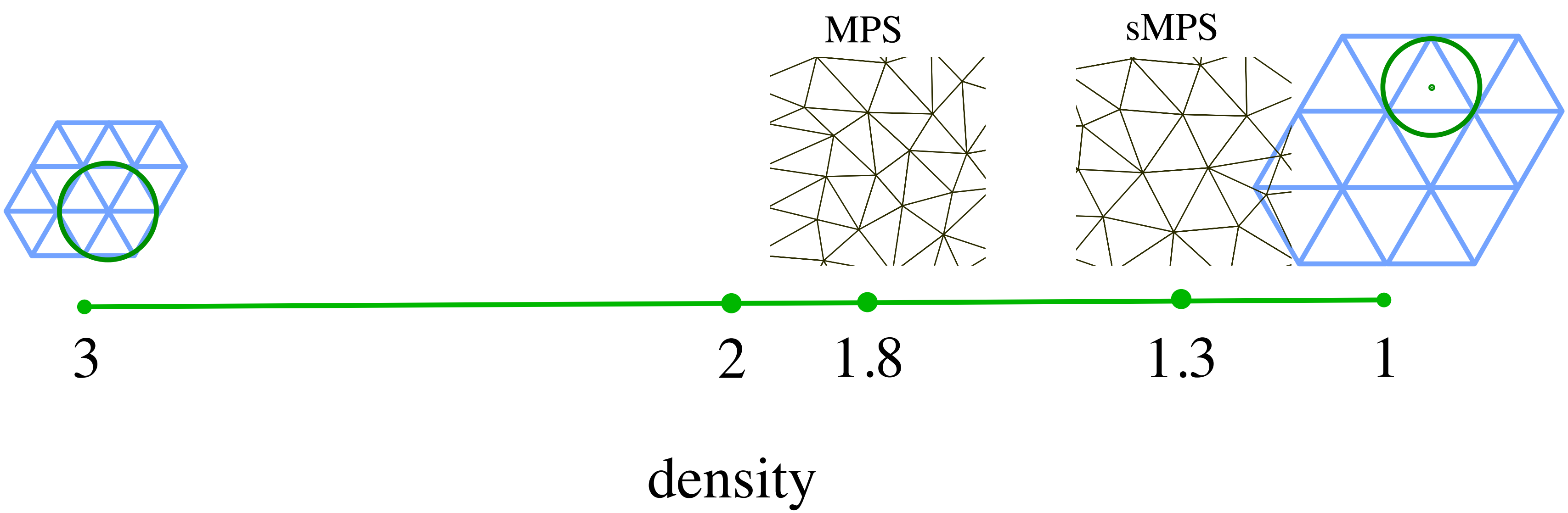
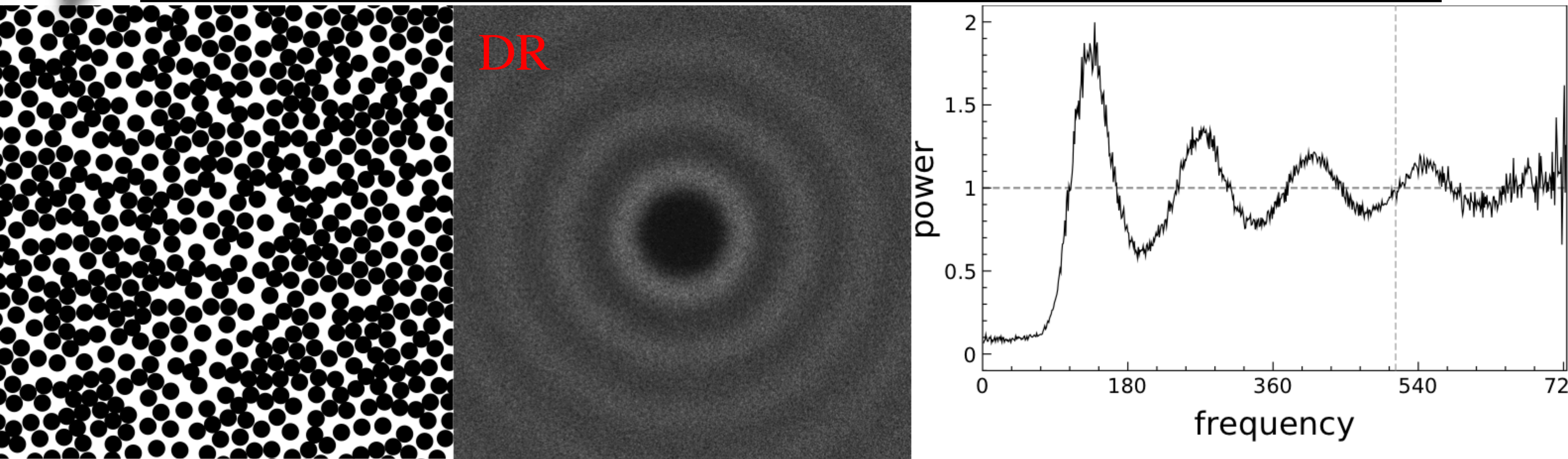
What's happening?

- Gets less dense but never gets close to "converging" to a structured mesh
  - No pair can be replaced by one.
  - A triple can be replaced by two? Would we want to?



Sifting (introduces?) Randomness
Surprise! But not identical.

DR



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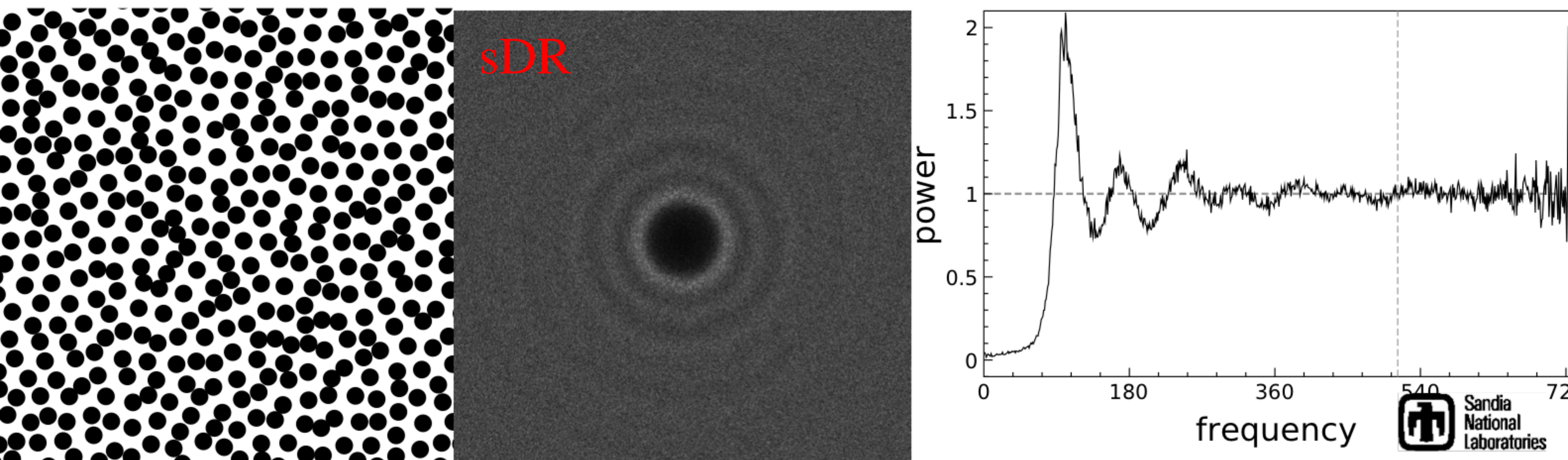

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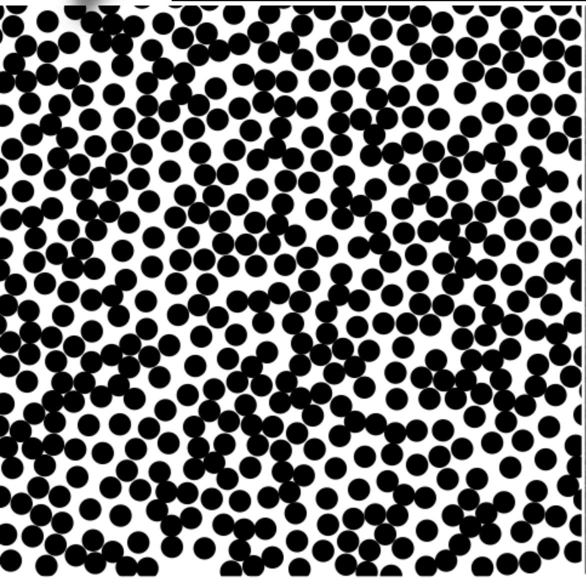
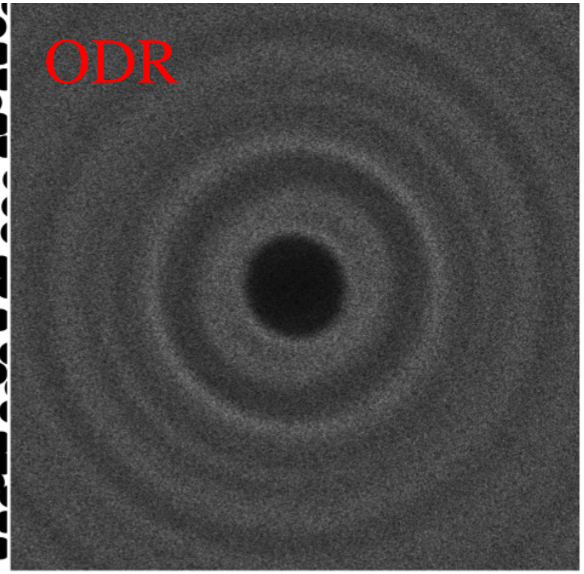
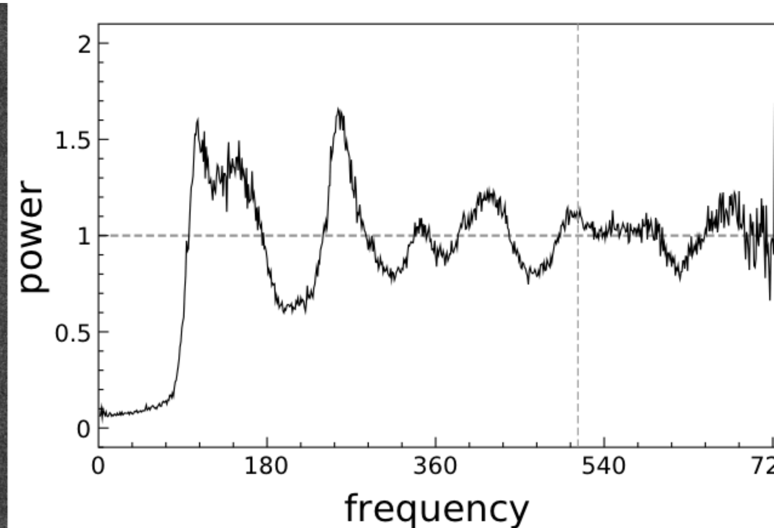
Sifting (introduces?) Randomness
Surprise! But not identical.

ODR







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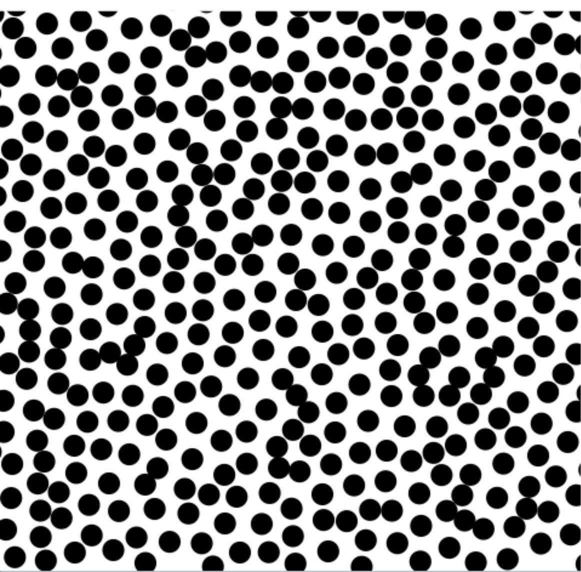
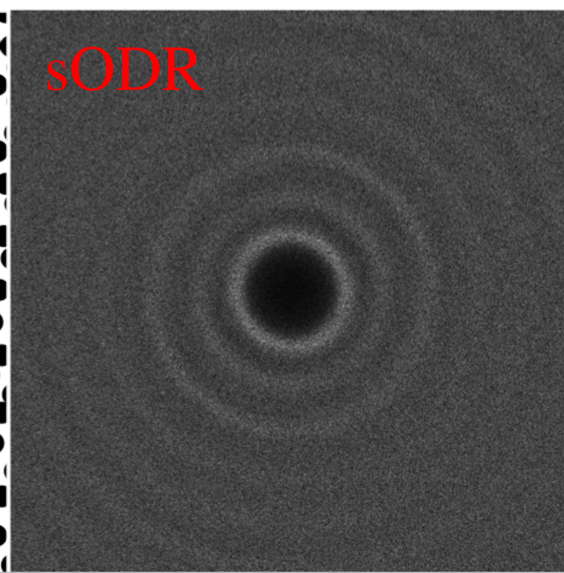
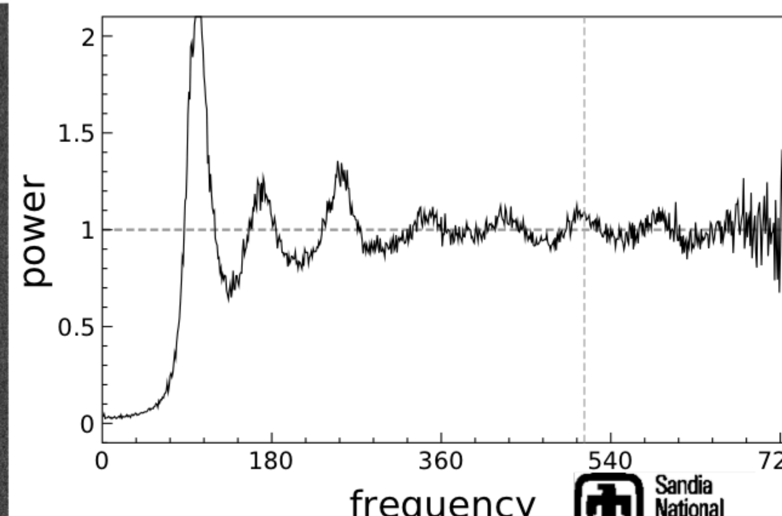
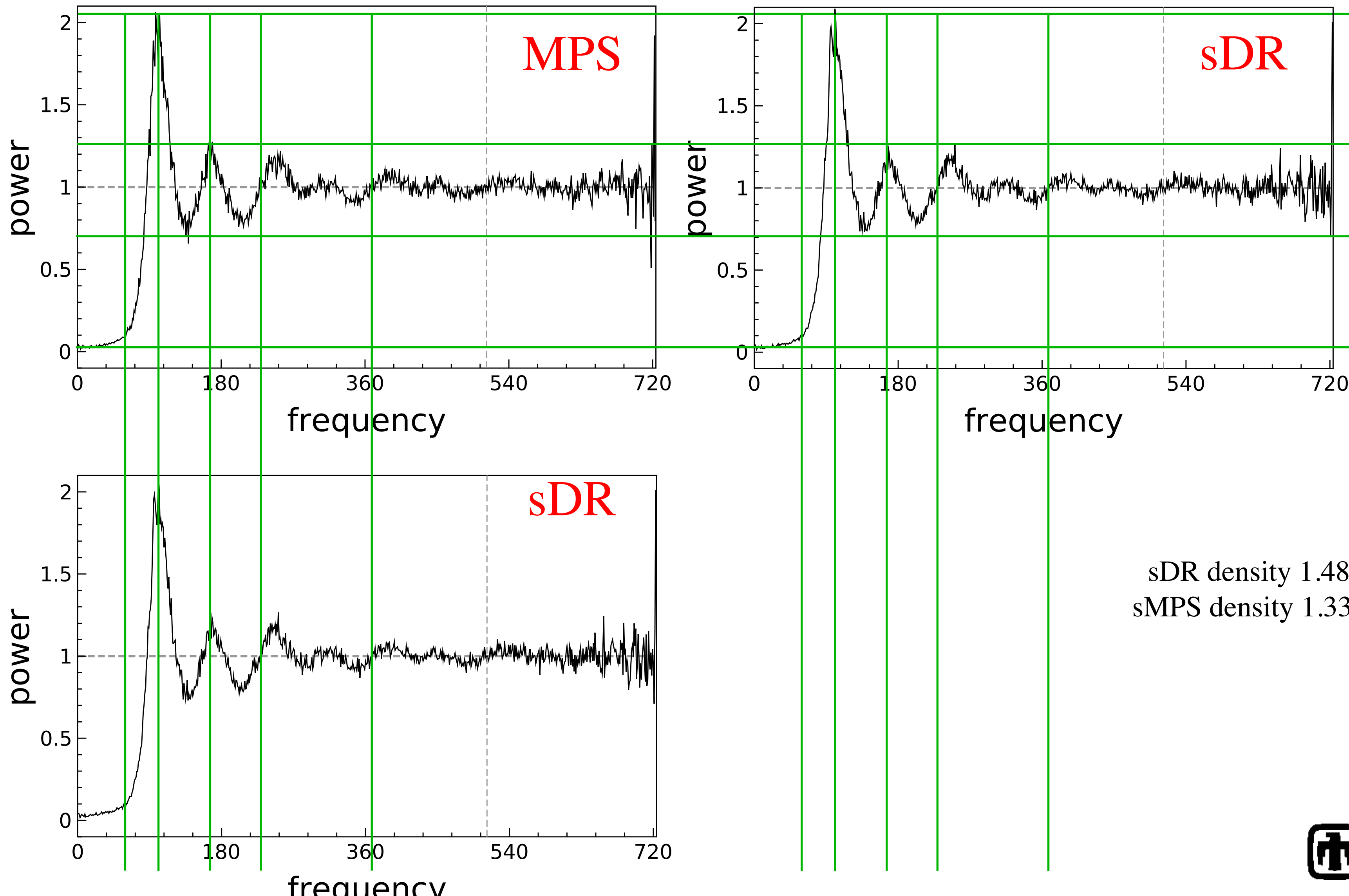
What’s ideal?







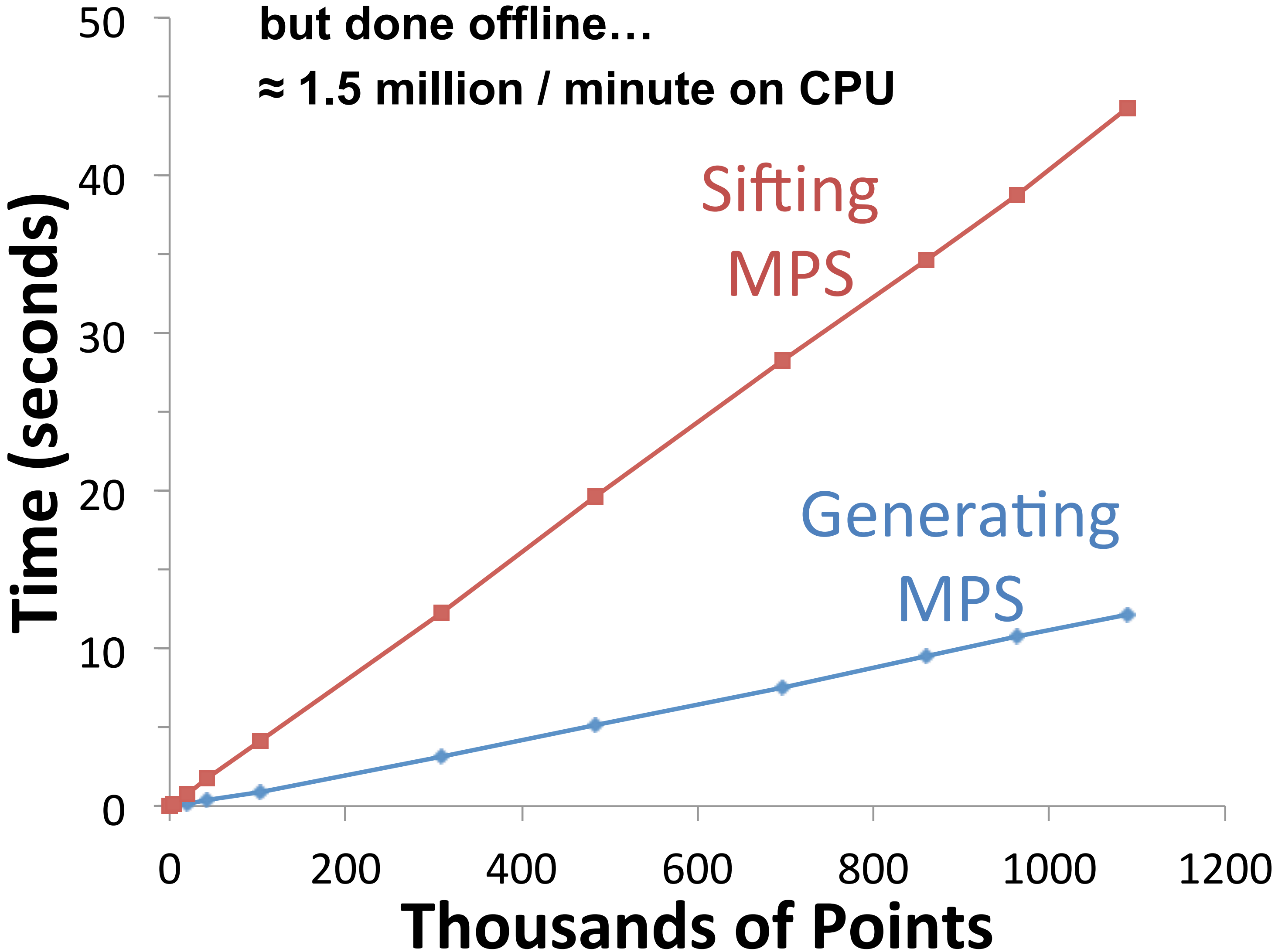
Original distribution doesn't seem to matter much



sDR density 1.48
sMPS density 1.33

Time and Memory Effectively Linear

- **Sifting 4x slower than generating MPS but done offline…**
  ≈ 1.5 million / minute on CPU



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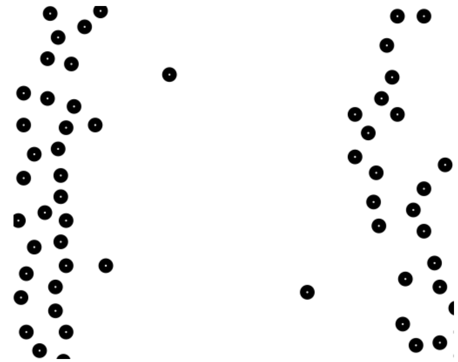
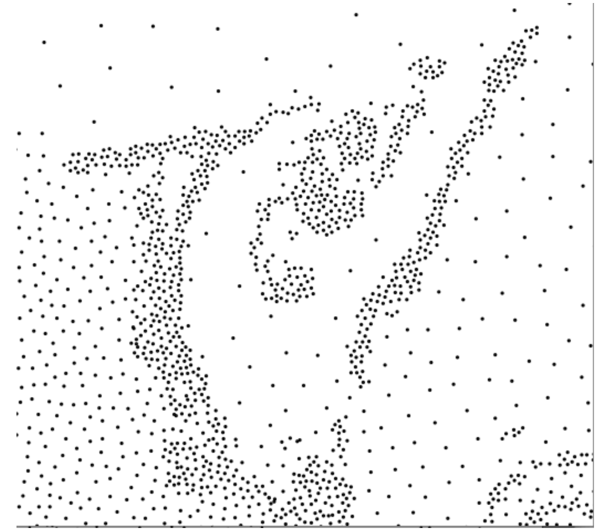
Beyond Uniform

- Prior was all 2d, constant radius
  - Spatially varying radii
    - Theory
      - Maximum rate of change L
    - Stippling application
      - L exceeded, still works





grayscale sizing function, high contrast

abrupt density changes

Beyond 2d

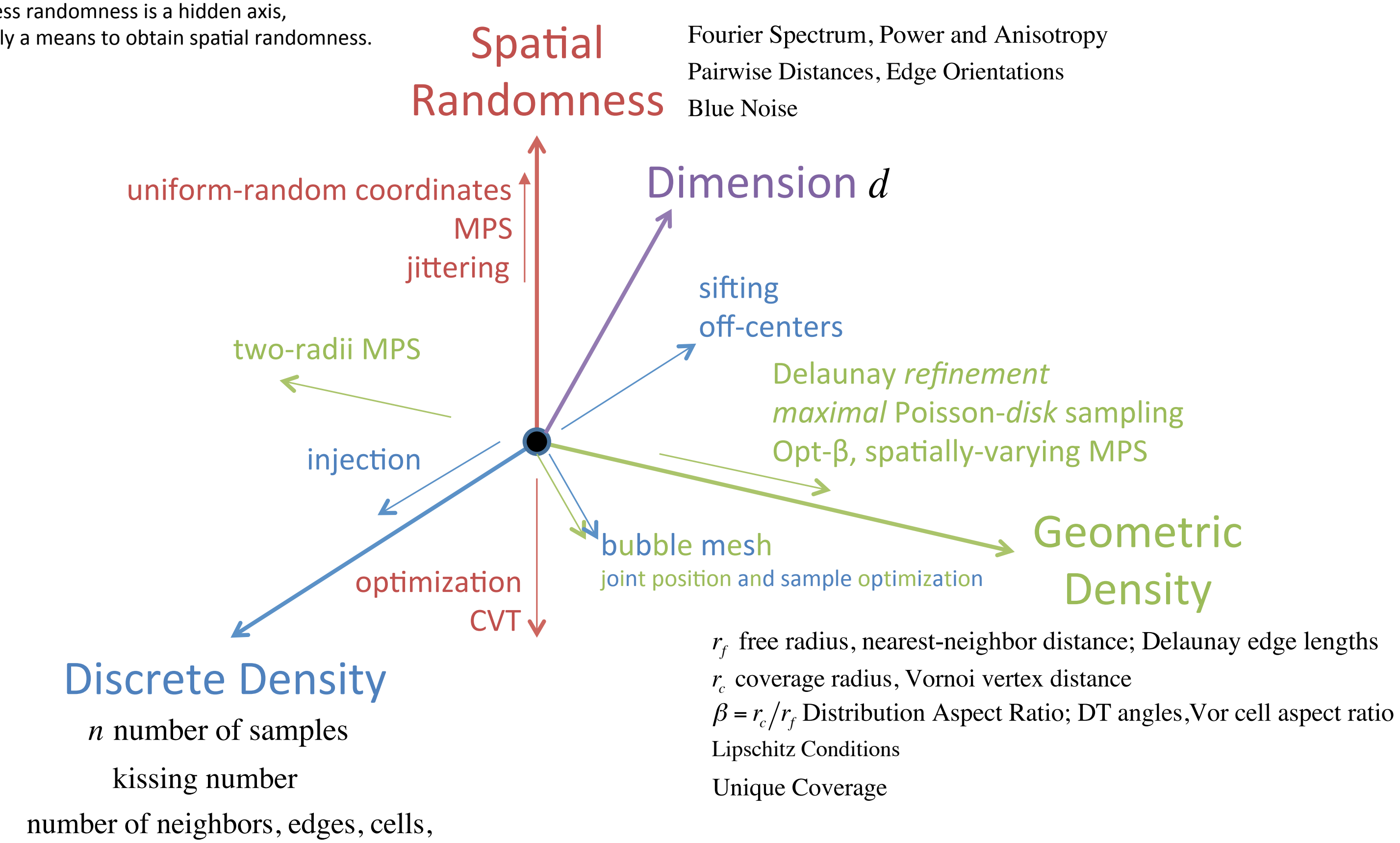
- Prior was all 2d, constant disk radius
  - Higher dimension
    - Seems straightforward to implement
    - effectiveness unknown

Bonus Thought
how I think about sampling

Scott: reorganized this so metrics are in black, the concept being measured in large colored text, and techniques in smaller non-cap text.
One can imagine both local and global measures, min max ave dev, for each of the axes and their metrics, some more natural than others.

A Space for All Sampling Methods

Process randomness is a hidden axis,
merely a means to obtain spatial randomness.



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Summary

- Sifting (replace 2-for-1) points
  - Reduces the number of points
  - Retains randomness and quality
  - Poisson-disk sampling as a subroutine - resample
- To do
  - Theory for rapidly varying sizing function, L >> 1
  - High dimensions
  - Generate a sparser distribution to begin with